

2. A biologist investigating the shell size of turtles takes random samples of adult female and adult male turtles and records the length, x cm, of the shell. The results are summarised below.

	Number in sample	Sample mean \bar{x}	$\sum x^2$
Female	6	19.6	2308.01
Male	12	13.7	2262.57

You may assume that the samples come from independent normal distributions with the same variance.

The biologist claims that the mean shell length of adult female turtles is 5 cm longer than the mean shell length of adult male turtles.

- (a) Test the biologist’s claim at the 5% level of significance. **(10)**
- (b) Given that the true values for the variance of the population of adult male turtles and adult female turtles are both 0.9 cm^2 ,
- (i) show that when samples of size 6 and 12 are used with a 5% level of significance, the biologist’s claim will be accepted if $4.07 < \bar{X}_F - \bar{X}_M < 5.93$ where \bar{X}_F and \bar{X}_M are the mean shell lengths of females and males respectively.
 - (ii) Hence find the probability of a type II error for this test if in fact the true mean shell length of adult female turtles is 6 cm more than the mean shell length of adult male turtles. **(6)**



Leave
blank

Question 2 continued

Ruled area for writing the answer to Question 2.



P 4 0 4 7 3 A 0 5 2 4

6. When a tree seed is planted the probability of it germinating is p . A random sample of size n is taken and the number of tree seeds, X , which germinate is recorded.

(a) (i) Show that $\hat{p}_1 = \frac{X}{n}$ is an unbiased estimator of p .

(ii) Find the variance of \hat{p}_1 .

(4)

A second sample of size m is taken and the number of tree seeds, Y , which germinate is recorded.

Given that $\hat{p}_2 = \frac{Y}{m}$ and that $\hat{p}_3 = a(3\hat{p}_1 + 2\hat{p}_2)$ is an unbiased estimator of p ,

(b) show that

(i) $a = \frac{1}{5}$,

(ii) $\text{Var}(\hat{p}_3) = \frac{p(1-p)}{25} \left(\frac{9}{n} + \frac{4}{m} \right)$.

(6)

(c) Find the range of values of $\frac{n}{m}$ for which

$$\text{Var}(\hat{p}_3) < \text{Var}(\hat{p}_1) \text{ and } \text{Var}(\hat{p}_3) < \text{Var}(\hat{p}_2)$$

(3)

(d) Given that $n=20$ and $m=60$, explain which of \hat{p}_1 , \hat{p}_2 or \hat{p}_3 is the best estimator.

(3)



Leave
blank

Question 6 continued

A series of horizontal lines for writing, intended for the continuation of Question 6.



P 4 0 4 7 3 A 0 1 9 2 4